Computation and Modeling Assignment 24

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Problem 24-2

Suppose we have a coin that lands on heads with probability k and tails with probability $1-k$. We flip the coin 5 times and get HHTTH.

1. Compute the likelihood of the observed outcome if the coin were fair (i.e. $k = 0.5$).

   Solution:

   $$\begin{aligned} P(\text{HHTTH}|k=0.5) &= P(\text{H}|k=0.5)P(\text{H}|k=0.5)P(\text{T}|k=0.5)P(\text{T}|k=0.5)P(\text{H}|k=0.5) \\ &= (0.5)^5 \\ &= 0.03125 \end{aligned}$$

2. Compute the likelihood of the observed outcome if the coin were slightly biased towards heads, say $k = 0.55$.

   Solution:

   $$\begin{aligned} P(\text{HHTTH}|k=0.5) &= P(\text{H}|k=0.55)P(\text{H}|k=0.55)P(\text{T}|k=0.55)P(\text{T}|k=0.55)P(\text{H}|k=0.55) \\ &= (0.55)(0.55)(0.45)(0.45)(0.55) \\ &= (0.55)^3(0.45)^2 \\ &= 0.03369 \end{aligned}$$

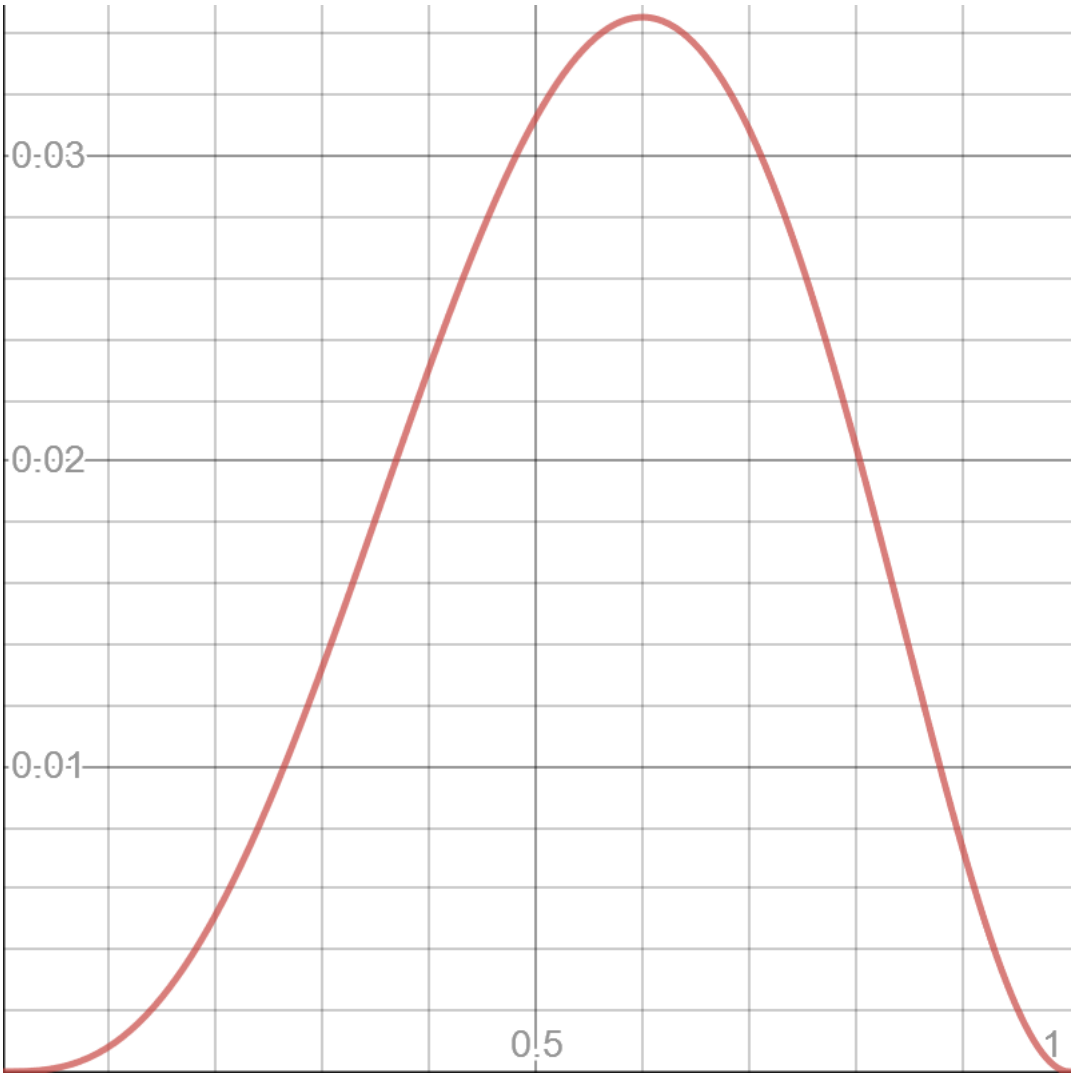
3. Compute the likelihood of the observed outcome for a general value of p. Your answer should be a function of k.

   Solution:

   $$\begin{aligned} P(\text{HHTTH}|k) &= P(\text{H}|k)P(\text{H}|k)P(\text{T}|k)P(\text{T}|k)P(\text{H}|k) \\ &= k^3(k-1)^2 \end{aligned}$$

4. Plot a graph of $P(HHTTH|k)$ for $0 \leq k \leq 1$.

   Solution:


0.03
0.02
0.01
0.5
1