Eurisko Assignment 28-1

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Suppose you are given the following dataset:

$$\text{data} = [(1, 0.2), (2, 0.25), (3, 0.5)]$$

Fit a linear regression model $y = a + bx$ **by hand** by

1. setting up a system of equations,
2. turning the system into a matrix equation,
3. finding the best approximation to the solution of that matrix equation by using the pseudoinverse, and
4. substituting your solution for the coefficients of the model.

Show all of your steps.

Solution

Using the data points that we have, we have the equations

$$\begin{aligned} a + b &= 0.2 \\ a + 2b &= 0.25 \\ a + 3b &= 0.5 \end{aligned}$$

Turning that into a matrix equation, we get

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.25 \\ 0.5 \end{bmatrix}$$

The pseudoinverse of a matrix A is $(A^T A)^{-1} A^T$.

In our case,

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$

Therefore, the pseudoinverse of A is

$$
\begin{aligned}
(A^T A)^{-1} A^T &= \left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \\
&= \frac{1}{3 \cdot 14 - 6 \cdot 6} \begin{bmatrix} 14 & -6 \\ -6 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \\
&= \frac{1}{6} \begin{bmatrix} 8 & 2 & -4 \\ -3 & 0 & 3 \end{bmatrix}
\end{aligned}
$$

Multiplying both sides by the pseudoinverse, we get

$$
\begin{aligned}
\frac{1}{6} \begin{bmatrix} 8 & 2 & -4 \\ -3 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} &= \frac{1}{6} \begin{bmatrix} 8 & 2 & -4 \\ -3 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0.2 \\ 0.25 \\ 0.5 \end{bmatrix} \\
\frac{1}{6} \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} &= \frac{1}{6} \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix} \\
\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} &= \begin{bmatrix} 0.0166 \\ 0.15 \end{bmatrix} \\
\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} &= \begin{bmatrix} 0.0166 \\ 0.15 \end{bmatrix}
\end{aligned}
$$

Therefore,

$$
y = 0.0166 + 0.15x
$$