Machine Learning Assignment 46

Colby Roberts

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Several witnesses reported seeing a UFO during the following time intervals:

$$data = [[12, 13], [12, 13.5], [14, 15], [14, 16]]$$

The times represent hours in military time:

12 is noon,

13 is 1 pm,

13.5 is 1:30 pm,

Suppose you want to quantify your certainty regarding when the UFO arrived and when it left.

Assume the data came from U[a,b], the uniform distribution on the interval [a,b]. This means the UFO arrived at time a and left at time b.

Solutions

(A) The likelihood function, $\mathcal{L}([a,b]|\text{data})$ is:

$$\begin{aligned}\mathcal{L}([a,b]|\text{data}) &= \frac{13-12}{b-a} \cdot \frac{13.5-12}{b-a} \cdot \frac{15-14}{b-a} \cdot \frac{16-14}{b-a} \\ &= \frac{(1)\cdot(1.5)\cdot(1)\cdot(2)}{(b-a)^4} \\ &= \frac{3}{(b-a)^4}\end{aligned}$$

(B) Normalization of the likelihood function, $\mathcal{L}([a,b]|\text{data})$ is:

$$\begin{aligned}\int_{b_{min}}^{b_{max}} \int_{a_{min}}^{a_{max}} c \cdot \mathcal{L}([a,b]|\text{data})\, da\, db &= \int_{16} \int_{-}^{12} c \cdot \frac{3}{(b-a)^4}\, da\, db \\ &= \int_{16} c \cdot \frac{1}{(b-12)^3}\, db \\ &= c \cdot (\frac{1}{32} \\ &\implies c \cdot \frac{1}{32} = 1 \\ &\implies c = 32\end{aligned}$$

(C) the probability that the UFO came and left sometime during the day that it was sighted? In other words, the probability that $0 < a < a_{max}$ and $b_{min} < b < 24$ is:

$$\begin{aligned}
\int_{16}^{24}\int_{0}^{12} c\cdot\mathcal{L}([a,b]|\text{data}) &= \int_{16}^{24}\int_{0}^{12} 32\cdot\frac{3}{(b-a)^4}\,da\,db \\
&= 96\cdot\int_{16}^{24}\frac{1}{3(b-12)^3}-\frac{1}{3b^3}\,db \\
&= 96\cdot\frac{41}{4608} \\
&= \frac{41}{48}
\end{aligned}$$

(D) the probability that the UFO arrived before 10am is:

$$\begin{aligned}
\int_{16}^{24}\int_{0}^{12} c\cdot\mathcal{L}([a,b]|\text{data}) &= \int_{16}^{24}\int_{0}^{12} 32\cdot\frac{3}{(b-a)^4}\,da\,db \\
&= 32\cdot\int_{16}^{24}\frac{1}{3(b-10)^3}\,db \\
&= 32\cdot\frac{1}{72} \\
&= \frac{32}{72} \\
&= \frac{4}{9}
\end{aligned}$$