Machine Learning Assignment 58

Colby Roberts

November 6, 2020

60-1

60-1-A

By Big-O definition $f(n) \leftarrow (3n^2 + 2n + 1)$ is $O(n^2)$ if $T(n) \leq c \cdot n^2$ for some $n \geq n_0$. Let us check this condition: if $3n^2 + 2n + 1 \leq c \cdot n^2$ then $3 + \frac{2}{n} + \frac{1}{n^2} \leq c$. Therefore the Big-O condition holds for $n \geq n_0 = 1$ and $c \geq 6(= 3 + 2 + 1)$. Larger values of n_0 result in similar factors c but in any case the previous statement is valid.

60-1-B

Assume there is a function $h(n)$ such that $h(n)$ is $O(f(n) + g(n))$ if and only if there is an M such that $h(n) \leq M(f(n) + g(n))$ for large enough n, which is equivalent to $h(n) \leq max(M\{f(n), g(n)\}) + min(M\{f(n), g(n)\})$ for large enough n. For that same M the following would then also be true. Here we depend on the assumption that both f(n) and g(n) are positive for large n. $h(n) \leq max(2M\{f(n), g(n)\})$ for large enough n. If we now choose P to be 2M, then we can say we have a P for which $h(n) \leq max(P\{f(n), g(n)\})$ for large enough n. which according to the definition of big-O means that $h(n)$ is $O(max\{f(n), g(n)\})$

60-1-C

Assume there is a function $f_1(n) = O(g_1(n)$ and $f_2(n) = O(g_2(n))$. We must show that $f_1(n) \cdot f_2(n) = O(g_1(n) \cdot g_2(n))$. Let $f_1(n) \leq c_1 \cdot g_1(n)$ for $n \geq n_0$. We repeat the same process for $f_2(n)$. So let $f_1(n) \cdot f_2(n) \leq (c_1 \cdot g_1(n)) \cdot (c_2 \cdot g_2(n))$. This implies that $f_1(n) \cdot f_2(n) = O(g_1(n) \cdot g_2(n))$

60-1-D

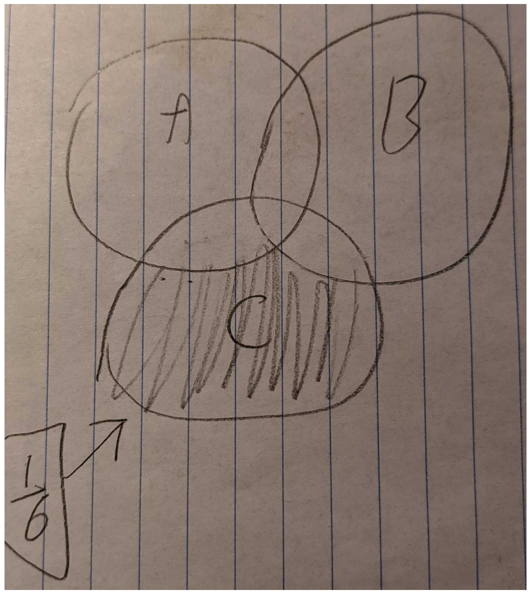
We need to define that $g(n) = O(g(n))$, so we can prove $f(n) = O(g(n))$, $g(n) = O(h)$, and $f(n) = O(h(n))$. We can use the previous proofs, so let c be some constant in $\mathbb{R}$. Let $g(n) = g(n) \implies g(n) = c * g(n) \implies g(n) = O(g(n))$ for $n \geq n_0$. Because we know that $g(n) = O(g(n))$ we say that $f(n) = O(g(n)) = g(n) = O(h) \implies f(n) = O(h)$

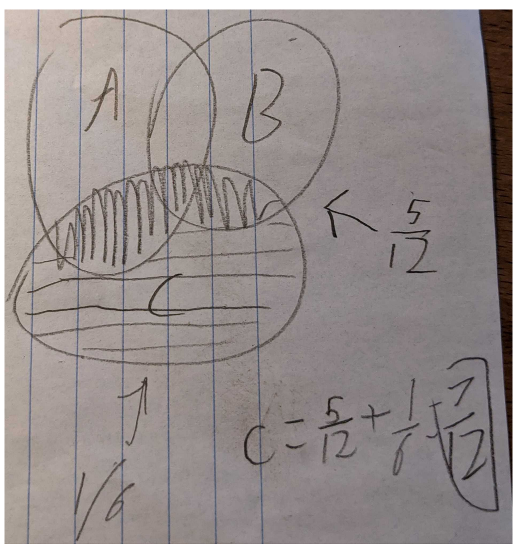
60-2

60-2-A

$$A: P(A \cap B) = P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$

B : Yes, because the union of all three sections make the whole space S, making them by definition partitions of S

C : 

D : 

60-2-B

$$\begin{aligned}
P(T<3|T>2) &= P(T\leq 3 \mid T\geq 2)\\
&= \frac{P(T\leq 3\cap T>2}{P(T>2)}\\
&= \frac{P(2<T\leq 3)}{P(T>2)}\\
&= \frac{e^{-2/5}-e^{-3/5}}{e^{-2/5}}\\
&= 0.18126924692
\end{aligned}$$

60-2-C

We can define the lengths of the triangle to be x, $y-x$, and $1-y$, so we consider the that three segments of length a, b, and c form a triangle in general, so we must satisfy the inequalities of triangles,

$$\begin{aligned}
a+b&>c\\
b+c&>a\\
c+a&>b
\end{aligned}$$

So applying this to our current situation we have,

$$\begin{aligned}
x+(y-x)&>1-y\\
(y-x)+(1-y)&>x\\
(1-y)+x&>y-x
\end{aligned}$$

Simplified we get,

$$\begin{aligned}
y&>\frac{1}{2}\\
x&<\frac{1}{2}\\
y&<x+\frac{1}{2}
\end{aligned}$$

When we graph these inequalities we get an area of $\frac{1}{8}$, so this gives us the probability of forming a triangle where x ¡ y, and because of symmetry its the same for x ¿ y, so,

$$\begin{aligned}
P(\text{forming a triangle}) &= P(\text{forming a triangle}|x<y)+P(\text{forming a triangle}|x>y)\\
&= \frac{1}{8}+\frac{1}{8}\\
&= 1/4
\end{aligned}$$

60-3

60-3-A

i : False, because the gender coefficient is positive with the female predictor being ¿ 0 and the male predictor = 0

ii : True, see i

iii : False, because the GPA coefficient and predictor is small so it wont cover the 35 thousand dollar deficit that the males have under the females, also the females will get the same boost from the GPA as well.

iv : True see iii

60-3-B

$$
\begin{aligned}
&\beta_0 + 4 \cdot \beta_1 + 110 \cdot \beta_2 + 1 \cdot \beta_3 + 440 \cdot \beta_4 + 4 \cdot \beta_5 \\
&= 50 + 4 \cdot 20 + 110 \cdot 0.07 + 1 \cdot 35 + 440 \cdot 0.01 + 4 \cdot -10 \\
&= \$137.1 \text{ Thousand Dollars}
\end{aligned}
$$

60-3-C

False, because even if the interaction coefficient is very small, this is to counteract the massive interaction term, an example of this would be this regression where the interaction coefficient is 0.01, but the interaction term is 440, increasing the result by 4.4, this in regression terms is $4,400 which is a lot.