Problem 47-1

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$$Data = [12, 13], [12, 13.5], [14, 15], [14, 16]$$

1 Compute the likelihood function L([a,b]|data)

$$\frac{13-12}{b-a} * \frac{13.5-12}{b-a} * \frac{15-14}{b-a} * \frac{16-14}{b-a} = \frac{1.5*2}{(b-a)^4} = \frac{3}{(b-a)^4}$$

2 Normalize the likelihood function so that it can be interpreted as a probability density.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c * \frac{3}{(b-a)^4} dadb = 1$$

Because of the earliest and latest arrivals:

$$c\int_{16}^{\infty}\int_{-\infty}^{12} c * \frac{3}{(b-a)^4} dadb = 1$$

$$c\int_{16}^{\infty} \frac{1}{(b-12)^3} - \frac{1}{\infty} db = c * \frac{-1}{2*(b-12)^3}\Big|_{16}^{\infty} = c*(0+\frac{1}{32}) = 1$$

c=$\frac{1}{32}$=1 so c=32 so the valid probability is:

$$\frac{96}{(b-a)^4}$$

3 What is the probability that the UFO came and left sometime during the day that it was sighted? In other words, what is the probability that $0 < a < a_{max}$ and $b_{min} < b < 24$?

$$\int_{16}^{24}\int_{0}^{12} \frac{96}{(b-a)^4} dadb = \int_{16}^{\infty} \frac{32}{(b-12)^3} - \frac{32}{b^3} db =$$

$$\frac{16}{b^2} - \frac{16}{(b-12)^2}\Big|_{16}^{24} = \frac{16}{24^2} - \frac{16}{12^2} - \frac{16}{16^2} + \frac{16}{4^2} = \frac{41}{48}$$

4 What is the probability that the UFO arrived before 10am?

$$\int_{16}^{\infty}\int_{-\infty}^{10} \frac{96}{(b-a)^4} da db = \int_{16}^{\infty} \frac{32}{(b-10)^3} db = \frac{-16}{(b-10)^2}\Big|_{16}^{\infty} = \frac{16}{36} = \frac{4}{9}$$