Machine Learning Assignment 47

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Problem 1

(a) *Compute the likelihood function $\mathcal{L}([a,b]|data)$*

$$\begin{aligned}
\mathcal{L}([a,b]|\text{data}) &= \mathcal{L}([a,b]|[12,13],[12,13.5],[14,15],[14,16]) \\
&= \frac{13-12}{b-a} * \frac{13.5-12}{b-a} * \frac{15-14}{b-a} * \frac{16-14}{b-a} \\
&= \frac{3}{(b-a)^4}
\end{aligned}$$

(b) *Normalize the likelihood function so that it can be interpreted as a probability density.*

$$\int_{b_{\min}}^{b_{\max}} \int_{a_{\min}}^{a_{\max}} c * \mathcal{L}([a,b]|\text{data}) \mathrm{d}a \, \mathrm{d}b$$

The absolute earliest the UFO could have left is at 16, because that was the latest sight of it. The latest it could have arrived is at 12, which is the earliest it was sighted. This means that $a_{\max} = 12$ and $b_{\min} = 16$. We aren't sure when it arrived or left, and it isn't guaranteed that it arrived or left on that same day, so we can set the other bounds to infinity. We have $a_{\min} = -\infty$ and $b_{\max} = \infty$.

Therefore, we have

$$
\begin{aligned}
1 &= \int_{16}^{\infty} \int_{-\infty}^{12} c * \frac{3}{(b-a)^4} \mathrm{d}a \, \mathrm{d}b \\
&= c * \int_{16}^{\infty} -\frac{1}{(b-a)^3} \Big|_{-\infty}^{12} \mathrm{d}b \\
&= c * \int_{16}^{\infty} -\frac{1}{(b-12)^3} \mathrm{d}b \\
&= -c * \frac{1}{2(b-12)^2} \Big|_{16}^{\infty} \\
&= -c * \frac{-1}{32} \\
c &= 32
\end{aligned}
$$

Now we can plug this into the probability density and get:

$$
p([a,b]) = \frac{96}{(b-a)^4}
$$

(c) *What is the probability that the UFO came and left sometime during the day that it was sighted? In other words, what is the probability that* $0 < a < a_{max}$ *and* $b_{min} < b < 24$*?*

$$
\begin{aligned}
&\int_{16}^{24} \int_{0}^{12} \frac{96}{(b-a)^4} \mathrm{d}a \, \mathrm{d}b \\
&= \int_{16}^{24} \frac{32}{(b-a)^3} \Big|_{0}^{12} \mathrm{d}b \\
&= \int_{16}^{24} \frac{32}{(b-12)^3} - \frac{32}{b^3} \mathrm{d}b \\
&= \frac{16}{(b-12)^2} - \frac{16}{b^2} \Big|_{16}^{24} \\
&= \frac{-5}{144} - \frac{-8}{9} \\
&= \frac{41}{48}
\end{aligned}
$$

(d) *What is the probability that the UFO arrived before 10am?*

$$
\begin{aligned}
&\int_{16}^{\infty} \int_{-\infty}^{10} \frac{96}{(b-a)^4} \mathrm{d}a \, \mathrm{d}b \\
&\qquad = \int_{16}^{\infty} -\frac{32}{(b-a)^3} \bigg|_{-\infty}^{10} \mathrm{d}b \\
&\qquad = \int_{16}^{\infty} -\frac{32}{(b-10)^3} \mathrm{d}b \\
&\qquad = -\frac{16}{(b-10)^2} \bigg|_{16}^{\infty} \\
&\qquad = \frac{16}{36} = \frac{4}{9}
\end{aligned}
$$