Machine Learning Assignment 50

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February 24, 2021

Problem 1

(0) *Given the data, compute the likelihood P(two rolls of 1 and three rolls of 2 |switch)*

$$P(\text{2 rolls of 1 \& 3 rolls of 2} = \begin{cases} \binom{5}{2} * \left(\frac{1}{2}\right)^5, \text{switch} = \text{True} \\ \binom{5}{2} * \left(\frac{1}{6}\right)^5, \text{switch} = \text{False} \end{cases}$$

(1a) *Given your prior belief, what is the prior distribution P(switch)?*

$$P(\text{switch}) = \begin{cases} 50\%, \text{switch} = \text{True} \\ 50\%, \text{switch} = \text{False} \end{cases}$$

(1b) *What is the posterior distribution P(switch |two rolls of 1 and three rolls of 2)?*

$$\begin{aligned} P(\text{switch } | \text{2 rolls of 1 and 3 rolls of 2 }) &= 1 \\ &= c\left(\binom{5}{2} * \left(\frac{1}{2}\right)^5 * 0.5 + \binom{5}{2} * \left(\frac{1}{6}\right)^5 * 0.5\right) \\ &= c\left(\frac{5}{32} + \frac{5}{7776}\right) \\ c &= \sim 6.374 \end{aligned}$$

$$P(\text{switch } | \text{2 rolls of 1 and 3 rolls of 2 }) = \begin{cases} \sim 0.996, \text{switch} = \text{True} \\ \sim 0.004, \text{switch} = \text{False} \end{cases}$$

(2a) *Given your prior belief, what is the prior distribution P(switch)?*

$$P(\text{switch}) = \begin{cases} 1\%, \text{switch} = \text{True} \\ 99\%, \text{switch} = \text{False} \end{cases}$$

(2b) *What is the posterior distribution P(switch |two rolls of 1 and three rolls of 2)?*

$$
\begin{aligned}
P(\text{switch } | 2 \text{ rolls of } 1 \text{ and } 3 \text{ rolls of } 2\ ) = 1 \\
&= c\left(\binom{5}{2} * \left(\frac{1}{2}\right)^5 * 0.01 + \binom{5}{2} * \left(\frac{1}{6}\right)^5 * 0.99\right) \\
&= c\binom{5}{2}\left(\left(\frac{1}{2}\right)^5 * 0.01 + \left(\frac{1}{6}\right)^5 * 0.99\right) \\
c =& \sim 227.37
\end{aligned}
$$

$$
P(\text{switch } | 2 \text{ rolls of } 1 \text{ and } 3 \text{ rolls of } 2\ ) = \begin{cases} \sim 0.711, \text{switch} = \text{True} \\ \sim 0.289, \text{switch} = \text{False} \end{cases}
$$

(3a) *Given your prior belief, what is the prior distribution P(switch)?*

$$
P(\text{switch}) = \begin{cases} 99\%, \text{switch} = \text{True} \\ 1\%, \text{switch} = \text{False} \end{cases}
$$

$$
\begin{aligned}
P(\text{switch } | 2 \text{ rolls of } 1 \text{ and } 3 \text{ rolls of } 2\ ) = 1 \\
&= c\left(\binom{5}{2} * \left(\frac{1}{2}\right)^5 * 0.99 + \binom{5}{2} * \left(\frac{1}{6}\right)^5 * 0.01\right) \\
&= c\binom{5}{2}\left(\left(\frac{1}{2}\right)^5 * 0.99 + \left(\frac{1}{6}\right)^5 * 0.01\right) \\
c =& \sim 3.232
\end{aligned}
$$

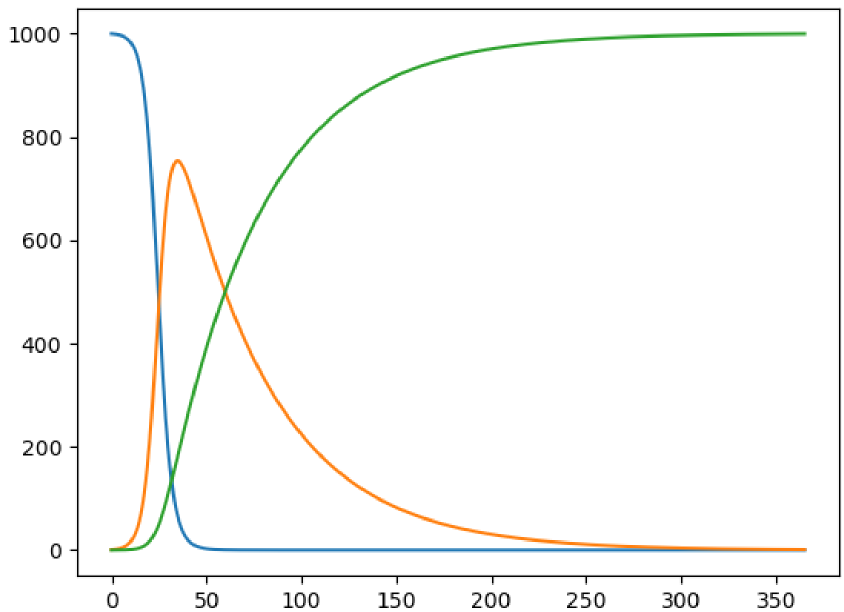
$$
P(\text{switch } | 2 \text{ rolls of } 1 \text{ and } 3 \text{ rolls of } 2\ ) = \begin{cases} \sim 0.99996, \text{switch} = \text{True} \\ \sim 0.00004, \text{switch} = \text{False} \end{cases}
$$

Problem 2

(a) *Write a system of differential equations to model the system.*

$$
\begin{cases} \frac{\mathrm{d}S}{\mathrm{d}t} = -0.0003SI, S(0) = 1000 \\ \frac{\mathrm{d}I}{\mathrm{d}t} = 0.0003SI - 0.02I, I(0) = 1 \\ \frac{\mathrm{d}R}{\mathrm{d}t} = 0.02I, R(0) = 0 \end{cases}
$$

(b) *Plot the system and include the plot in your Overleaf document.*



(c) *Explain what the plot shows, and explain why this happens.*

The plot shows the amount of susceptible people, the amount of infected people, and the amount of recovered people over time. Initially the susceptible starts out as the highest because only 1 person has the virus and it hasn't spread. (No one has recovered yet.) The infected people start meeting 10 people a day (because 1% of 1000 people) and only infect 3% of the people they meet. This makes the infected people go up exponentially. But, people can also recover. 2% of everyone who has the infection recover every day, so the infected people start going down and the recovered people start going up, making a logistic looking curve. It approaches 1001 recovered people, which is the entirety of the people in the simulation.