Machine Learning Assignment 60

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Problem 1

A.

According to the definition, we have:

$$\begin{aligned} \lim_{x\to\infty} \frac{3n^2+2n+1}{n^2} &< \infty \\ &= 3+0+0 < \infty \end{aligned}$$

B.

$$\begin{aligned} f(n)+g(n) &< 2\max(f,g) \\ O(f+g) &< c*\max(f,g) \\ O(f+g) = \max(f,g) & \\ \max(f,g) &< f+g \\ O(\max(f,g)) &= f+g \\ O(f+g) = O(\max)(f+g) & \end{aligned}$$

C.

$$\begin{aligned} O(g) &< cg \\ O(f) &< cf \\ O(f)+O(g) &< c(f\dot{g}) \\ O(f)+O(g) &= O(f\dot{g}) \end{aligned}$$

D.

$$
\begin{aligned}
f &< cg \\
g &< ch \\
fg &< cgh \\
f &< ch \\
f &= O(h)
\end{aligned}
$$

Problem 2

A.

a.

$$
\begin{aligned}
&\frac{1}{2} + \frac{2}{3} - 2x = \frac{5}{6} \\
&x = \frac{1}{6}
\end{aligned}
$$

b.

True. $A \cup B \cup C = S$

c.

$$1 - \frac{5}{6} = \frac{1}{6}$$

d.

$$1 - \frac{5}{6} + \frac{5}{12} = \frac{7}{12}$$

B.

$$\frac{P(T \geq 2) - P(T \geq 3)}{P(T \geq 2)} = \frac{0.6703 - 0.5488}{0.6703} = 0.1813$$

C.

If we picture it geometrically like the image provided, we will notice that each edge of the triangle is $\sqrt{2}$ units long, according to Pythagorean's theorem ($1^2 + 1^2 = c^2$). This means that the area of the whole triangle is $a^2\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$. Now, the region in this one in which all points make valid triangles, can be modeled as an upside-down equilateral triangle, similar to the larger one but with edge lengths half as long. This will make the new edge lengths $\frac{\sqrt{2}}{2}$, which makes the area of the inner triangle $\frac{\sqrt{3}}{8}$. Since we are trying to find the probability, dividing the area of the valid region by the area of the possible region will give us the percent the broken stick can be made into a valid triangle.

Problem 3

a.

$$\begin{aligned}
x_1 &= \text{some constant gpa} \\
x_2 &= \text{some constant IQ} \\
\text{male model} &= 50 + 20x_1 + 0.07x_2 + 0.01x_1x_2 \\
\text{female model} &= 85 + 10x_1 + 0.07x_2 + 0.01x_1x_2
\end{aligned}$$

i. This is false, because there are multiple combinations of numbers in which the male can actually earn less than women. As explained in (iii), if the GPA is lower than 3.5, the man will earn less.

ii. This is false, for a similar reason to (i). If the GPA is higher than 3.5, men will have a higher salary.

iii. True. With a GPA higher than 3.5, men will earn more than females. We can prove this by setting the male salary as greater than the female salary, and solve for the range of GPA's which make that true. We have:

$$\begin{aligned}
50 + 20x_1 + 0.07x_2 + 0.01x_1x_2 &> 85 + 10x_1 + 0.07x_2 + 0.01x_1x_2 \\
10x_1 &> 35 \\
x_1 &> 3.5
\end{aligned}$$

This means that as long as the GPA is higher than 3.5, the men will have a higher salary, given a constant IQ.

iv. False. For the reasons mentioned in (iii), females will earn less than men.

b.

$50 + 20 * 4.0 + 0.07 * 110 + 35 * 1 + 0.01 * 110 * 4.0 - 10 * 4.0 * 1 = 137.1\text{k/year}$

c.

This is false. GPA is on a range from 0 to 4, and IQ goes from 0 to around 160 (though technically there is no limit). If a genius walked in with a GPA of 4 and an IQ of 160, the IQ/GPA interaction term would turn out to be: $0.01 * 160 * 4$ which turns out to be 6.4. This is actually a significant amount that you can add to your salary just by being a genius, and for those with an IQ of over 160, they will get an even larger salary from this term.