Part A

Solution

We just have to multiply the probability of each one happenin The function $p(x, y)$ is nonzero only for $x \in [a, b] \times [c, d]$, so we replace the bounds and integrand accordingly and solve for k as follows:

$$\frac{13-12}{b-a} \cdot \frac{13.5-12}{b-a} \cdot \frac{15-14}{b-a} \cdot \frac{16-14}{b-a}$$

$$\frac{1}{b-a} \cdot \frac{1.5}{b-a} \cdot \frac{1}{b-a} \cdot \frac{2}{b-a}$$

$$\frac{3}{(b-a)^4}$$

Part B

Solution

We have to double integrate from negative infinity to infinity, the likelihood function we calculated above multiplied by normalizing constant, c. And we have to set it all equal to 1

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c \cdot \frac{3}{(b-a)^4} dadb = 1$$

$$\int_{16}^{\infty} \int_{-\infty}^{12} c \cdot \frac{3}{(b-a)^4} dadb$$

Move the c completely out of the integral for now, compute the first integral

$$\int_{-\infty}^{12} \frac{3}{(b-a)^4} da = \frac{1}{(b-12)^3}$$

Compute the second integral

$$\int_{16}^{\infty} \frac{1}{(b-12)^3} = \frac{1}{32}$$

Calculate c

$$\frac{c}{32} = 1$$
$$c = 32$$

Normalized Likelihood Function:

$$\frac{96}{(b-a)^4}$$

Part C

Solution

Double integral from 0 to 24 hours, using the amax of 12 and bmin of 16

$$\int_{16}^{24}\int_{0}^{12}\frac{96}{(b-a)^4}dadb$$

Solve first integral

$$\int_{0}^{12}\frac{96}{(b-a)^4}da = \frac{32}{(b-12)^2} - \frac{32}{b^3}$$

Second integral

$$\int_{16}^{24}\frac{32}{(b-12)^3} - \frac{32}{b^3}db = \frac{41}{48}$$

Part D

Solution

Same process as before but with amax as 10 and from -infinity to infinity

$$\int_{16}^{\infty}\int_{-\infty}^{10}\frac{96}{(b-a)^4}dadb$$

First integral

$$\int_{-\infty}^{10}\frac{96}{(b-a)^4}da = \frac{32}{(b-10)^3}$$

Second Integral

$$\int_{16}^{\infty}\frac{32}{(b-10)^3}db = \frac{4}{9}$$