Machine Learning Assignment 44

Your Name

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Problem 1

(a) Solution

$$p(x) = \begin{cases} \frac{1}{1-0} \in [0,1]\\ 0 \notin [0,1] \end{cases}$$
$$E[x] = \int_0^1 x * p(x)$$
$$E[x] = \frac{1}{2}$$
$$E[x^2] = \int_0^1 x^2 * p(x)$$
$$\int_0^1 x^2 = \frac{x^3}{3} \Big|_0^1$$
$$E[x^2] = \frac{1}{3}$$
$$Cov[X, X^2] = E[(X - \frac{1}{2})(X^2 - \frac{1}{3})]$$
$$E[X^3 - \frac{X}{3} - \frac{x^2}{2} - \frac{1}{6}]$$

Separate

$$E[X^3] - \frac{1}{3}E[X] - \frac{1}{2}E[x^2] - \frac{1}{6}$$
$$E[x^3] = \int_0^1 x^3 * p(x)$$

$$E[x^3] = \frac{1}{4}$$

Back to former equation

$$\frac{1}{4} - \frac{1}{6} - \frac{1}{6} - \frac{1}{6}$$

I got $-\frac{1}{12}$. Which is wrong but I don't know what I did wrong (b)

Solution

$$E[X_1] = \frac{1}{2}$$

$$E[X_2] = \frac{1}{2}$$

$$Cov[X_1, X_2] = E[(X_1 - \frac{1}{2}))(X_2 - \frac{1}{2})]$$

$$E[X_1 * X_2 - \frac{X_2}{2} - \frac{X_1}{2} + \frac{1}{4}]$$

$$\frac{1}{4} - \frac{1}{4} - \frac{1}{4} + \frac{1}{4} = 0$$

(c)

Solution

$$Var[X_1 + X_2] = E[(X_1 + X_2)^2] - E[X_1 + X_2]^2$$
$$X_1^2 + 2X_1 * X_2 + X_2^2 - (X_1^2 + 2X_1 * X_2 + X_2^2)$$
$$X_1^2 - X_1^2 + X_2^2 - X_2^2 + 2(X_1 * X_2 - X_1 * X_2)$$

 $Var(X_1) + Var(X_2) + 0$

Did I go about this the wrong way? (d)

Solution

$$Cov[X_1, X_2] = E[(X_1 - E[x_1])(X_2 - E[X_2])]$$
$$E[X_1 * X_2 - E[X_2] * X_1 - E[X_1] * X_2 + E[X_1] * E[X_2]]$$

 $E[X_1 * X_2] - E[X_1] * E[X_2] - E[X_1] * E[X_2] + E[X_1] * E[X_2]$

 $E[X_1 * X_2] - E[X_1] * E[X_2]$