Distribution Properties

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Problem 33-2

a. *Given that $X \sim p(x)$, where $p(x)$ is a continuous distribution, prove that for any real number a we have $E[aX] = aE[X]$.*

b. *Given that $X_1, X_2 \sim p(x)$, where $p(x)$ is a continuous distribution, prove that $E[X_1 + X_2] = E[X_1] + E[X_2]$.*

c. *Given that $X \sim p(x)$ where $p(x)$ is a continuous probability distribution, prove the identity $Var[X] = E[X^2] - E[X]^2$.*

Solution

a.

$$
\begin{aligned}
\mathrm{E}[aX] &= \int_{-\infty}^{\infty} a \cdot X \cdot p_\lambda(x)\,\mathrm{d}x \\
&= a \cdot \int_{-\infty}^{\infty} X \cdot p_\lambda(x)\,\mathrm{d}x \\
&= a \cdot \mathrm{E}[X]
\end{aligned}
$$

b.

$$
\begin{aligned}
\mathrm{Var}[X] &= \mathrm{E}[(X - \mathrm{E}[X])^2] \\
&= \mathrm{E}[X^2 - 2X\mathrm{E}[X] + \mathrm{E}[X]^2] \\
&= \mathrm{E}[X^2] - 2\mathrm{E}[X\mathrm{E}[X]] + \mathrm{E}[\mathrm{E}[X]^2] \\
&= \mathrm{E}[X^2] - 2\mathrm{E}[X]\mathrm{E}[X] + \mathrm{E}[X]^2 \\
&= \mathrm{E}[X^2] - 2\mathrm{E}[X]^2 + \mathrm{E}[X]^2 \\
&= \mathrm{E}[X^2] - \mathrm{E}[X]^2
\end{aligned}
$$

(used $\mathrm{E}[\mathrm{E}[X]] = \mathrm{E}[X]$)