20-1

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$$p_2(x) = \begin{cases} 2e^{-2x} x \geq 0 \\ 0 \quad x < 0 \end{cases}$$

Problem a

Using integration, show that this is a valid distribution, i.e. all the probability integrates to 1.

$$\begin{aligned} \int_{-\infty}^{\infty} p_2(x)dx &= 0 + \int_0^{\infty} 2e^{-2x}dx \\ &= -\frac{1}{e^{2x}} - (-e^0) \\ &= \frac{1}{\infty} + 1 \\ &= 1 \end{aligned}$$

Problem b

Given that $X \sim p_2$, compute $\mathbf{P}(0 < X \leq 1)$.

$$\begin{aligned} P(0 < X \leq 1) &= \int_0^1 p_2(x)dx \\ &= \int_0^1 2e^{-2x}dx \\ &= -e^{-2} - (-1) \\ &= 1 - e^{-2} \end{aligned}$$

Problem c

Given that $X \sim p_2$, compute $E[X]$.

$$\begin{aligned}
E[X] &= \int_{-\infty}^{\infty} x p_2(x) dx \\
&= 0 + \int_{0}^{\infty} 2x e^{-2x} dx \\
&= -\frac{\infty}{e^{\infty}} - (-\frac{e^0}{2}) \\
&= \frac{1}{2}
\end{aligned}$$

Problem d

Given that $X \sim p_2$, compute Var$[X]$.

$$\begin{aligned}
Var[X] &= \int_{-\infty}^{\infty} (x - \frac{1}{2})^2 p_2(x) dx \\
&= \int_{0}^{\infty} (x^2 - x + \frac{1}{4}) 2e^{-2x} dx \\
&= \int_{0}^{\infty} 2x^2 e^{-2x} - 2x e^{-2x} + \frac{e^{-2x}}{2} dx \\
&= -\frac{\infty}{4e^{\infty}} - (-\frac{1}{4e^0}) \\
&= \frac{1}{4}
\end{aligned}$$