33-2

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Problem a

Given that $X \sim p(x)$, where $p(x)$ is a continuous distribution, prove that for any real number a we have $E[aX] = aE[X]$.

The expected value of a continuous distribution goes as follows:

$$E[X] = \int_{-\infty}^{\infty} x \cdot p(x) dx$$

Substituting ax for x we get:

$$E[aX] = \int_{-\infty}^{\infty} (ax) \cdot p(x) dx$$

Since a is a constant, we can pull it out of the integral:

$$E[aX] = a \int_{-\infty}^{\infty} x \cdot p(x) dx$$
$$E[aX] = a \cdot E[X]$$

Problem b

Given that $X \sim p(x)$ where $p(x)$ is is a continuous probability distribution, prove the identity $\mathbf{Var}[X] = E[X^2] - E[X]^2$.

$$
\begin{aligned}
Var[X] &= E[(X - E[X])^2] \\
&= \int_{-\infty}^{\infty} (X - E[X])^2 \cdot p(x)dx \\
&= \int_{-\infty}^{\infty} (X^2 - 2XE[X] + E[X]^2) \cdot p(x)dx \\
&= \int_{-\infty}^{\infty} X^2 \cdot p(x)dx - \int_{-\infty}^{\infty} 2XE[X] \cdot p(x)dx + \int_{-\infty}^{\infty} E[X]^2 \cdot p(x)dx \\
&= E[X^2] - 2E[XE[X]] + E[E[X]^2] \\
&= E[X^2] - 2E[X]E[X] + E[X]^2 \\
&= E[X^2] - 2E[X]^2 + E[X]^2 \\
&= E[X^2] - E[X]^2
\end{aligned}
$$