Machine Learning Assignment 44

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$$data = \{[12, 13], [12, 13.5], [14, 15], [14, 16]\}$$

Part A

Compute the likelihood function for $\boldsymbol{L}([a, b]|data)$:

$$\boldsymbol{L}([a, b]|data) = P([12, 13]) * P(12, 13.5) * P([14, 15]) * P([14, 16])$$
$$\frac{13 - 12}{b - a} \cdot \frac{13.5 - 12}{b - a} \cdot \frac{15 - 14}{b - a} \cdot \frac{16 - 14}{b - a} = \frac{1 \cdot 1.5 \cdot 1 \cdot 2}{(b - a)^4}$$
$$= \frac{3}{(b - a)^4}$$

Part B

Normalize the probability distribution of $\boldsymbol{L}([a, b]|data)$
We know that $\boldsymbol{L}([a, b]|data) = \frac{3}{(b-a)^4}$. And this is a valid probability distribution if

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c \cdot \frac{3}{(b - a)^4} dadb = 1$$

This integral can be reduced to the following because the distribution is based off of data and we can base it off of the latest arrival and earliest departure

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{3}{(b - a)^4} dadb = \int_{16}^{\infty} \int_{-\infty}^{12} \frac{3}{(b - a)^4} dadb =$$

$$\int_{16}^{\infty} \frac{1}{(b - 12)^3} - \frac{1}{\infty} db = \int_{16}^{\infty} \frac{1}{(b - 12)^3} db = \left. \frac{-1}{2(b - 12)^2} \right|_{16}^{\infty} = 0 + \frac{1}{32}$$

So we need $c \cdot \frac{1}{2} = 1$ so c=32 and the valid probability distribution is

$$\frac{96}{(b - a)^4}$$

Part C

What is the probability that the UFO came and left sometime during the day that it was sighted?
The UFO being seen in the day means that $0 < a <$amax and bmin$< b < 24$ We also know that amax = 14 and bmin = 13. So we need to find the pro.babilty that $0 < a < 12$ and $16 < b < 24$ This can be represented as

$$\int_{16}^{24} \int_{0}^{12} \frac{96}{(b-a)^4} dadb = \int_{16}^{24} \frac{32}{(b-12)^3} - \frac{32}{b^3} db =$$

$$\frac{16}{b^2} - \frac{16}{(b-12)^2}\bigg|_{13}^{24} = \frac{16}{24^2} - \frac{16}{12^2} - \frac{16}{13^2} + \frac{1}{1^2} = \frac{41}{48}$$

Part D

What is the probability that the UFO arrived before 10am?
The probability that the UFO arrived before 10am is the probability that a¡10. with that in mind our limits become $16 < b < \infty$ and $-\infty < a < 10$. So we set up the integral as:

$$\int_{16}^{\infty} \int_{-\infty}^{10} \frac{96}{(b-a)^4} dadb = \int_{16}^{\infty} \frac{32}{(b-10)^3} db = \frac{-16}{(b-10)^2}\bigg|_{16}^{\infty} = \frac{16}{36} = \frac{4}{9}$$