Assignment 49-1

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Part A.

Given that X∼p(x), where p(x) is a continuous distribution, prove that for any real number a we have $E[\,aX] = aE[\,X]\,$.
Since p(x) is a continuous distribution then where X∼p(x) then

$$E[X] = \int_{-\infty}^{\infty} x \cdot p(x)dx$$

and

$$E[aX] = \int_{-\infty}^{\infty} ax \cdot p(x)dx = a\int_{-\infty}^{\infty} x \cdot p(x)dx = aE[X]$$

Therefore E[ax] = aE[X]

Part B

Given that X1,X2∼p(x), where p(x) is a continuous distribution, prove that $E[\,X1 + X2] = E[\,X1] + E[\,X2]\,$.

$$E[X_1 + X_2] = \int_{-\infty}^{\infty} (x_1 + x_2)p(x)dx = \int_{-\infty}^{\infty} x_1p(x) + x_2p(x)dx$$

$$\int_{-\infty}^{\infty} x_1p(x)dx + \int_{-\infty}^{\infty} x_2p(x)dx = E[X_1] + E[X_2]$$

Part C

Given that X∼p(x) where p(x) is a continuous probability distribution, prove the identity $Var[\,X] = E[X^2] - E[X]^2$

$$Var[X] = \int_{-\infty}^{\infty} (x - E[X])^2p(x)dx = \int_{-\infty}^{\infty} (x^2 - 2xE[X] + E[X]^2)p(x)dx =$$

$$\int_{-\infty}^{\infty} x^2p(x)dx - 2E[X]\int_{-\infty}^{\infty} xp(x)dx + E[X]^2\int_{-\infty}^{\infty} p(x)dx =$$

$$E[X^2] - 2E[X]^2 + E[X]^2 = E[X^2] - E[X]^2$$

Part D

Use bisection search to estimate $\sqrt{5}$ to 4 decimal places by hand, showing your work at each step of the way.

For a bisection search we need to find a function whose root is 0 meaning if f(x) = 0 then x = $\sqrt{5}$. For our bisection search we will use the function $f(x) = x^2 - 5$. This function works because if$f(x) = 0 \Rightarrow x^2 - 5 = 0 \Rightarrow x^2 = 5 \Rightarrow x = \sqrt{5}$. Also we will search between 2 and 3 because $\sqrt{4} = 2$ and $\sqrt{9} = 3$. We will now start our bisection search.

interval	guess(midpoint)	f(guess)	new bounds
$[2, 3]$	2.5	f(2.5) $= 1.25 > 0$	$[2, 2.5]$
$[2, 2.5]$	2.25	f(2.25) $= 0.0625 > 0$	$[2, 2.25]$
$[2, 2.25]$	2.125	f(2.125) = -0.484 < 0	$[2.125, 2.25]$
$[2.125, 2.25]$	2.1875	f(2.1875) = -0.2148	$[2.1875, 2.25]$
$[2.1875, 2.25]$	2.21875	f(2.21875) = -0.0771	$[2.21875, 2.25]$
$[2.21875, 2.25]$	2.234375	f(2.234375) = -0.0075	$[2.234375, 2.25]$
$[2.234375, 2.25]$	2.2421875	f(2.2421875) = 0.0274	$[2.234375, 2.2421875]$
$[2.234375, 2.2421875]$	2.23828125	f(2.23828125) = 0.0099	$[2.234375, 2.23828125]$
$[2.234375, 2.23828125]$	2.236328125	f(2.236328125) = 0.0011	$[2.234375, 2.236328125]$
$[2.234375, 2.236328125]$	2.2353515625	f(2.2353515625) = -0.003	$[2.2353515625, 2.236328125]$
$[2.2353515625, 2.236328125]$	2.23583984375	f(2.23583984375) = -0.001	$[2.23583984375, 2.236328125]$
$[2.23583984375, 2.236328125]$	2.23608398438	f(2.23608398438) = 0.00007	$[2.23583984375, 2.23608398438]$
$[2.23583984375, 2.23608398438]$	2.23596191406	f(2.23596191406) = -0.00047	$[2.23596191406, 2.23608398438]$
$[2.23596191406, 2.23608398438]$	2.23602294922	f(2.23602294922) =-0.0002	$[2.23602294922, 2.23608398438]$

Now since both of our bounds start with a 2.2360 we know that $2.2360 < \sqrt{5} < 2.2361$

Part E

Use "merge sort" to sort the list [4,8,7,7,4,2,3,1]. Do the problem by hand and show your work at each step of the way.

A merge sort is a way of sorting a list by breaking it into its individual elements ans putting them back together in ordered pairs.Let us compute
$[4, 8, 7, 7, 4, 2, 3, 1] \rightarrow [4, 8, 7, 7], [4, 2, 3, 1] \rightarrow [4, 8], [7, 7], [4, 2], [3, 1]$
Now that we have pairs we will sort each pair and then combine sorted pairs
$[4, 8] \rightarrow [4], [8] \rightarrow [4, 8]$
$[7, 7] \rightarrow [7], [7] \rightarrow [7, 7]$
$[4, 2] \rightarrow [4], [2] \rightarrow [2, 4]$
$[3, 1] \rightarrow [3], [1] \rightarrow [1, 3]$
Now to combine the ordered pairs
$[4, 8], [7, 7] \rightarrow [4, 7, 7, 8]$
$[4, 2], [3, 1] \rightarrow [1, 2, 3, 4]$
Now to combine the larger lists
$[4, 7, 7, 8], [1, 2, 3, 4] \rightarrow [1, 2, 3, 4, 4, 7, 7, 8]$
And that is the sorted list!