Assignment 49-2

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1

(Bayesian inference with a uniform prior)
Start by assuming a uniform prior distribution (i.e. you know nothing about k until you've collected some data). So, k initially follows the uniform distribution over the interval [0,1]: $P(k) = \frac{1}{1-0} = 1, k[0,1]$

1.a

(1 point) Other people make 1000 successful trips through the wormhole with no failures. What is the likelihood function for k given these 1000 successful trips?

The likeliehood function for k is $L(k|1000 \text{ succeses})$ which is broken down to $[P(k)]^{1000} = k^{1000}$

1.b

(1 point) What is the posterior distribution for k given these 1000 successful trips? (This is the same as just normalizing the likelihood function). The posterior distribution for k will be ck for some c such that

$$\int_{-\infty}^{\infty} c \cdot L(k|1000succeses)dk = 1$$

This is the same as

$$c\int_{0}^{1} k^{1000}dk = c\,[\frac{k^{1001}}{1001}]_{0}^{1} = \frac{c}{1001} = 1, c = 1001$$

So the posterior distribution is $1001k^{1000}$

1.c

(2 points) Assuming that you will use the wormhole 500 times per year, what is the posterior probability that the risk of disappearing forever into the wormhole is no more than your risk of dying from a car crash in a given year (1 in 10000) To find the probability that you will fail assuming that you travel 500 times a year that is the same as finding $P(1 - k^{500}\frac{1}{10000}|1000 \text{ successes})$

We can evaluate this as

$$\int_{0.99999979999}^{1} 1001(k)^{1000}dk = (k)^{1001}{}_{0.99999979999}^{1} = 1-(0.99999979999)^{1001} = 0.0002$$

2

(Updating by inspecting the posterior)

You keep on collecting data until your posterior distribution is $P(k|?successes) =$ $5001k5000$. But then you forget how many successes you have counted. Because this is a rather simple scenario, it's easy to find this number by inspecting your posterior distribution.

2.a

(1 point) Looking at the given posterior distribution, how many successes have you counted?

I counted 5000 successes

2.b

(2 points) Suppose you observe 2000 more successes. What is the posterior distribution now?

$7001k^{7000}$

3

(Bayesian updating)

To get some practice with the general procedure of Bayesian inference with a non-uniform prior, let's re-do Part 2, supposing you weren't able to remember the number of successes by inspecting your posterior distribution.

This time, you'll use $P(k|?successes) = 5001k^{5000}$ as your prior distribution.

3.a

(3 points) Suppose you observe 2000 more successes. Fill in the blanks:

prior distribution:$P(k) = 5001k^{5000}$

likelihood: P(2000 more successes|k) = k^{2000}

prior ×likelihood: P(2000 more successes|k)P(k)=$k^{2000} \cdot 5001k^{5000}$

posterior distribution:$P(k|$2000 more successes)=$7001k^{7000}$

4

(Inference)

Let's go back to the moment when you forgot the number of successes, and your posterior distribution was is P(k—? successes)=5001k5000.

4.a

(3 points) Assuming that you will use the wormhole 500 times per year, how many more people do you need to observe successfully come out of the wormhole to be 99% sure the risk of disappearing forever into the wormhole is no more than your risk of dying from a car crash in a given year (1 in 10000)?

Similar to before we need to evaluate $P(1k^{500}\frac{1}{10000}|N$ more successes). But this time instead of finding that probability given that N = 1000 we are now finding N such that the probability is 0.99. This in integral form is

$$\int_{0.99999979999}^{1} (N+1)k^N dk = k^{N+1}|_{0.99999979999}^{1} = 1 - (0.99999979999)^{N+1}$$

Now we need to solve this equation for 0.99

$$1 - (0.99999979999)^{N+1} = 0.99 \rightarrow 0.01 = (0.99999979999)^{N+1} \rightarrow$$

$$\frac{0.01}{0.99999979999} = 0.99999979999^N \rightarrow 0.010000002 = 0.99999979999^N$$

$$N = 23024699$$