Machine Learning Assignment 50

Your Name

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Problem 1

(Part 0) *Given the data, compute the likelihood P(two rolls of 1 and three rolls of 2 | switch)?*

Solution

$$P(Switch) = \begin{cases} \binom{5}{3}\frac{1}{2}^5 = 0.3125, Switch = True \\ \binom{5}{3}\frac{1}{6}^5 = 0.001286, Switch = False \end{cases}$$

(Part 1.a) *Given your prior belief, what is the prior distribution P(switch)?*

Solution

$$P(Switch) = \begin{cases} \frac{1}{2}, Switch = True \\ \frac{1}{2}, Switch = False \end{cases}$$

(Part 1.b) *What is the posterior distribution P(switch|two rolls of 1 and three rolls of 2)?*

Solution

$$P(Switch) = \begin{cases} \frac{1}{2} * 0.3125 = 0.15625, Switch = True \\ \frac{1}{2} * 0.001286 = 0.000643, Switch = False \end{cases}$$

$$c * (0.15625 + 0.000643) = 1$$

$$c = \frac{1}{0.156893}$$
$$c = 6.37377$$

$$P(Switch) = \begin{cases} 0.15625 * 6.37377 = 0.996, Switch = True \\ 0.000643 * 6.37377 = 0.004, Switch = False \end{cases}$$

(Part 2.a) *Given your prior belief, what is the prior distribution P(switch)?*

Solution

$$P(Switch) = \begin{cases} \frac{1}{100}, Switch = True \\ \frac{99}{100}, Switch = False \end{cases}$$

(Part 2.b) *What is the posterior distribution P(switch|two rolls of 1 and three rolls of 2)?*

Solution

$$P(Switch) = \begin{cases} \frac{1}{100} * 0.3125 = 0.003125, Switch = True \\ \frac{99}{100} * 0.001286 = 0.00127314, Switch = False \end{cases}$$

$$c * (0.003125 + 0.00127314) = 1$$

$$c = \frac{1}{0.00439814}$$
$$c = 227.36884$$

$$P(Switch) = \begin{cases} 0.003125 * 227.36884 = 0.7105, Switch = True \\ 0.001286 * 227.36884 = 0.2924, Switch = False \end{cases}$$

(Part 3.a) *Given your prior belief, what is the prior distribution P(switch)?*

Solution

$$P(Switch) = \begin{cases} \frac{99}{100}, Switch = True \\ \frac{1}{100}, Switch = False \end{cases}$$

(Part 3.b) *What is the posterior distribution P(switch|two rolls of 1 and three rolls of 2)?*

Solution

$$P(Switch) = \begin{cases} \frac{99}{100} * 0.3125 = 0.309375, Switch = True \\ \frac{1}{100} * 0.001286 = 0.00001286, Switch = False \end{cases}$$

$$c * (0.309375 + 0.00001286) = 1$$

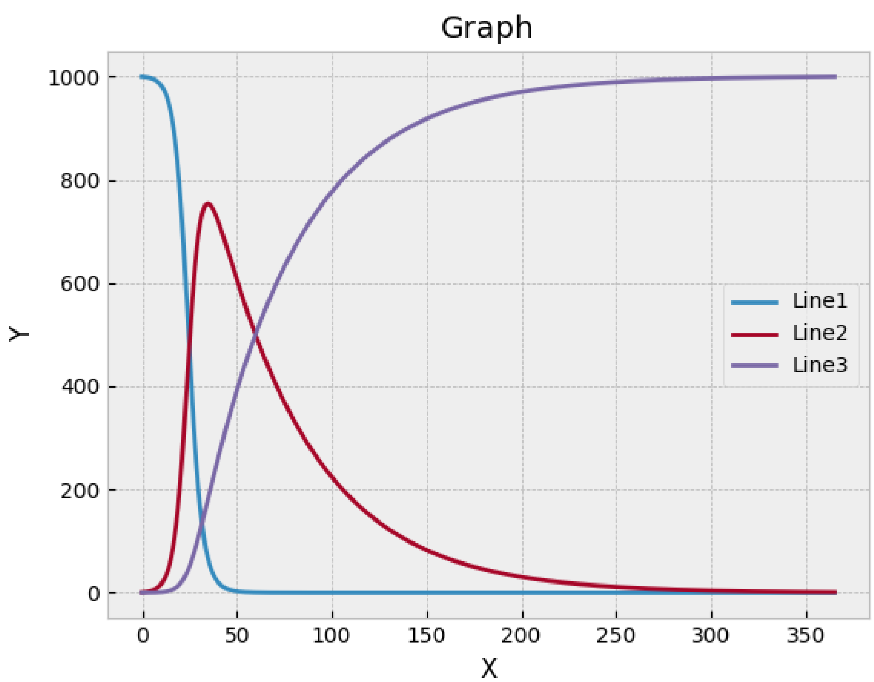
$$c = \frac{1}{0.30938786}$$
$$c = 3.23219$$

$$P(Switch) = \begin{cases} 0.309375 * 3.23219 = 0.99996, Switch = True \\ 0.00001286 * 3.23219 = 0.00004, Switch = False \end{cases}$$

Problem 2

(a) *Plot the system and include the plot in your Overleaf document.*

$$\begin{cases} \frac{\mathrm{d}S}{\mathrm{d}t} = (-0.0003)SI, \quad S(0) = 1000 \\ \frac{\mathrm{d}I}{\mathrm{d}t} = (0.0003)SI - 0.02I, \quad I(0) = 1 \\ \frac{\mathrm{d}R}{\mathrm{d}t} = (0.02)I, \quad R(0) = 0 \end{cases}$$



Line 1 is susceptible people, Line 2 is infected, and Line 3 is recovered.

The plot shows how a population of 1001 people changes with an infection, the susceptible people take a steep drop as more people get infected and the infected takes a steep spike. The reason the spike never reaches the original height of the susceptible people is because as much as people are getting infected, the rate at which people recover goes by as well. So the infected reaches it's peak and slowly drops as recovery increases and recovery takes a relatively steep increase and levels out as everyone recovers from the disease.