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1 Uniform Distribution

a. (2 points) Using the identity $Var[X] = E[X^2]E[X]^2$, compute Var[X] if X is sampled from the continuous uniform distribution U[a,b].

We first need to establish E[X] for U[a,b] = $\begin{cases} k & x \in [a,b] \\ 0 & x \notin [a,b] \end{cases}$

E[X] = $\int_a^b xkdx = \frac{kx^2}{2}\Big|_a^b = \frac{k}{2}(b^2 - a^2)$

$$Var[X] = k\frac{b^3 - a^3}{3} - \frac{k^2(b^2 - a^2)^2}{4}$$

Now we need to see if $E[X^2] - E[X]^2$ is the same as our Var[X].

$$E[X^2] = \int_a^b x^2 kdx = k\frac{x^3}{3}\Big|_a^b = k\frac{b^3 - a^3}{3}$$

$$E[X]^2 = \left(k\frac{b^2 - a^2}{2}\right)^2 = \frac{k^2(b^2 - a^2)^2}{4}$$

$$E[X^2] - E[X]^2 = k\frac{b^3 - a^3}{3} - \frac{k^2(b^2 - a^2)^2}{4}$$

This is the same as Var[X].

2 Exponential Distribution

b. (2 points) Using the identity $Var[X] = E[X^2]E[X]^2$, compute Var[X] if X is sampled from the exponential distribution p(x)=$\lambda e^{-\lambda x}, x \geq 0$.

Again we need to establish E[X]

$$E[X] = \int_0^\infty x\lambda e^{-\lambda x}dx = \left.\frac{-(\lambda x+1)e^{-\lambda x}}{\lambda}\right|_0^\infty = \frac{1}{\lambda}$$

$$Var[X] = \frac{1}{\lambda^2}$$

Now we need to find $E[X^2] - E[X]^2$

$$E[X^2] = \int_0^\infty x^2\lambda e^{-\lambda x}dx = -\left.\frac{(\lambda x(\lambda x+2)+2)e^{-\lambda x}}{\lambda^2}\right|_0^\infty = \frac{2}{\lambda^2}$$

$$E[X^2] - E[X]^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

This is the same as our Var[X]

3 Poisson Distribution

c. (2 points) Using the identity $Var[N] = E[N^2]E[N]^2$, compute Var[N] if N is sampled from the Poisson distribution p(n)=$\frac{\lambda^n e^{-\lambda}}{n!}, n \in \{0, 1, 2, \ldots\}$.

$$E[N] = \sum_{n=0}^{\infty}\frac{n\lambda^n e^{-\lambda}}{n!} = \sum_{n=0}^{\infty}\frac{\lambda^n e^{-\lambda}}{(n-1)!} = \frac{e^\lambda \cdot \lambda}{e^\lambda} = \lambda$$

$$Var[X] = \lambda$$

$$E[N^2] = \sum_{n=0}^{\infty}\frac{n^2\lambda^n e^{-\lambda}}{n!} = \sum_{n=0}^{\infty}\frac{n\lambda^n e^{-\lambda}}{(n-1)!} = \lambda^2 + \lambda$$

$$E[N^2] - E[N]^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$$

This is the same as our Var[X]