Assignment 60-2

Riley Paddock

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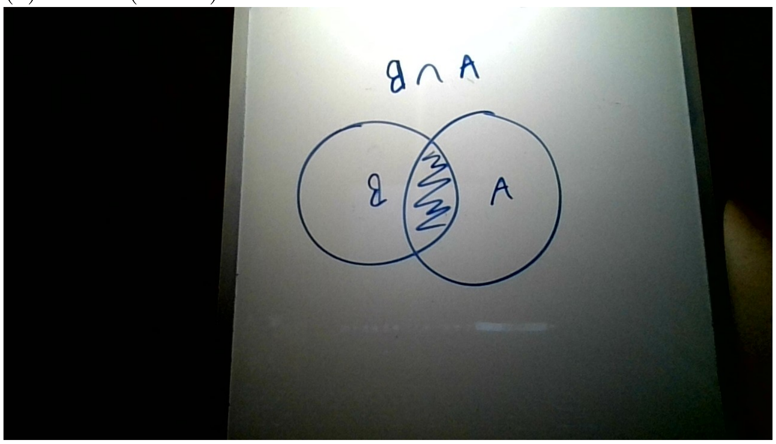
A.

- $A \cup B \cup C = S$
- P(A) $= \dfrac{1}{2}$
- P(B) $= \dfrac{2}{3}$
- P$(A \cup B) = \dfrac{5}{6}$

Answer the following.
(a) Find P$(A \cap B)$

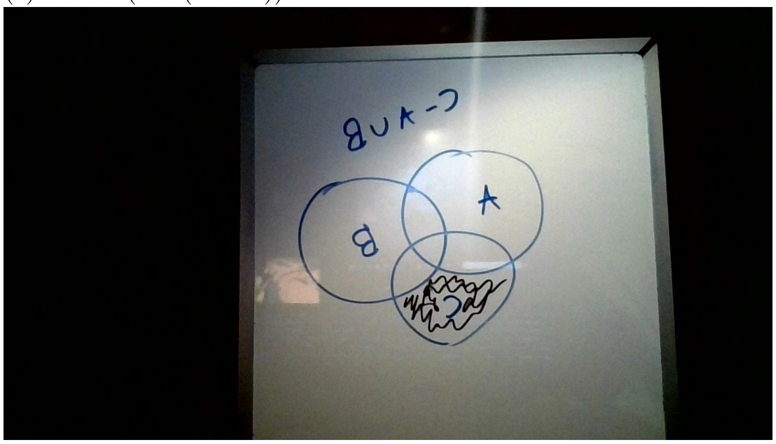


$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$P(A \cap B) = \frac{1}{2} + \frac{2}{3} - \frac{5}{6}$$
$$P(A \cap B) = \frac{3}{6} + \frac{4}{6} - \frac{5}{6}$$

$$P(A \cap B) = \frac{2}{6} = \frac{1}{3}$$

(b) Do A,B, and C form a partition of S
Yes A,B,C form a partition of S because together they contain all of S

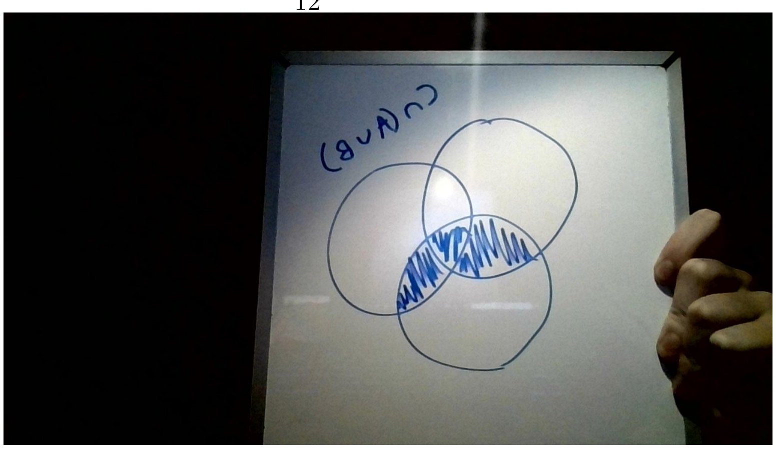
(c) Find P(C - $(A \cup B)$)



This is the same as $P(A \cup B \cup C - (A \cup B))$ because everything in $A \cup B \cup C$ that is not in $A \cup B$ is in C

$$P(A \cup B \cup C - (A \cup B)) = P(S - A \cup B) = P(S) - P(A \cup B) = 1 - \frac{5}{6} = \frac{1}{6}$$

(d) If P($C \cap (A \cup B)) = \frac{5}{12}$. find P(C)



$$P(C \cap (A \cup B)) = P(C) + P(A \cup B) - P(C \cup (A \cup B))$$
$$P(C) = P(C \cup (A \cup B)) - P(A \cup B) + \frac{5}{12}$$

$$P(C) = P(S) - \frac{5}{6} + \frac{5}{12}$$
$$P(C) = 1 - \frac{5}{12}$$
$$P(C) = \frac{7}{12}$$

B.

$$P(T \geq t) = e^{-\frac{t}{5}} \text{ for all t } \geq 0$$

What is the probability the tool will break down in the 3rd year given that it lasted 2 years.

$$P(t < 3 | t > 2) = \frac{P(t < 3 \cap t > 2)}{P(t > 2)} = \frac{P(2 < t < 3)}{P(t > 2)}$$

$$\frac{e^{-\frac{2}{5}} - e^{-\frac{3}{5}}}{e^{-\frac{2}{5}}} = 1 - e^{-\frac{1}{5}} = 0.1813.$$

C.

If we plot all the possibilities for the lengths of x,y,z in 3d space we get a triangle with vertices at (1,0,0),(0,1,0),(0,0,1). Then if we plot all the possibilities that can actually become triangles we have another triangle with vertex at (0.49,0.49,0.02),(0.49,0.02,0.49),(0.02,0.49,0.49). These vertex happen to be very close to the midpoints of the edges in the triangle of all possibilities. If we look at an equilateral triangle with a triangle formed within it by connecting the midpoints of each edge we have an equilateral triangle divided into 4 equal parts. Since only the middle part of this triangle gives us lengths that form a valid triangle that means the probability that the stick broken into 3 parts can form a triangle is approximately 1/4