Assignment 64-3

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part A

$$P_Y(k) = \begin{cases} Y = 0.7 & k = 0 \\ Y = 0.3 & k = 6 \end{cases}$$

part B

a)Find the range of the random variable R_x
The possibilities for R_x are [0,1,2]

b)Find $P(X \geq 1.5)$

$$P(X \geq 1.5) = 1 - P(X < 1.5) = 1 - (\frac{1}{2} + \frac{1}{6}) = \frac{2}{6} = \frac{1}{3}$$

c)Find $P(0 < X < 2)$

$$P(0 < X < 2) = P(X < 2) - P(X < 0) = \frac{5}{6} - 0 = \frac{5}{6}$$

d)Find $P(X = 0|X < 2)$

$$P(X = 0|X < 2) = \frac{P(X = 0 \cap X < 2)}{P(X < 2)} = \frac{\frac{1}{2}}{\frac{5}{6}} = \frac{6}{10}$$

part C

- A and C are independent
- B and C are independent
- A and B are disjoint

- P(A$\cup$ C) = $\frac{2}{3}$, P(B$\cup$ C) = $\frac{3}{4}$, P(A$\cup$ B$\cup$ C) = $\frac{11}{12}$

Find P(A), P(B), P(C)

P(A) + P(C) - P($A \cap B$) = $\frac{2}{3}$
P(B) + P(C) - P($A \cap C$) = $\frac{3}{4}$
P(A)+P(B)+P(C) - P($A \cap B$) - P($A \cap C$) = $\frac{11}{12}$
Adding the first 2 equations together we get:
P(A) + P(B) +2P(C) - P($A \cap B$) - P($A \cap C$) = $\frac{17}{12}$

Subtracting the third equation from that we get:
P(C) = $\frac{6}{12} = \frac{1}{2}$

Plugging back into the first equation and using A,C's independence we get:

$$P(A) + \frac{1}{2} - P(A)\frac{1}{2} = \frac{2}{3}$$

$$P(A)\left(1 - \frac{1}{2}\right) = \frac{2}{3} - \frac{1}{2}$$

$$P(A) = \frac{1}{3}$$

Plugging C into the second equation using B,C's independence and the same logic we get:

$$P(B)\left(1 - \frac{1}{2}\right) = \frac{3}{4} - \frac{1}{2}$$

$$P(B) = \frac{2}{4} = \frac{1}{2}$$

part D

The info given can be sorted into these equations:

P(A) + P(B) + P(C) + P(D) = 1
P(A) - P(B) = 0
P(C) - 2P(D) = 0
P(A) + P(C) = 0.6

As a matrix this is:

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 1 & 0 & 1 & 0 & 0.6 \end{array}\right]$$

Solving this we get

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{1}{5} \\ 0 & 1 & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 0 & 1 & \frac{1}{5} \end{array}\right]$$

This means:

$$\mathrm{P(A)} = \frac{1}{5}, \mathrm{P(B)} = \frac{1}{5}, \mathrm{P(C)} = \frac{2}{5}, P(D) = \frac{1}{5}$$