Machine Learning Assignment 44

Your Name

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Part 1

(a) *Write the probability distribution* $p_4(n)$ *for getting* n *heads on* 4 *coin flips, where the coin is a fair coin (i.e. it lands on heads with probability* 0.5*).*

Solution The probability distribution would be:

$$P(0) = 0.0625, P(1) = 0.25, P(2) = 0.375, P(3) = 0.25, P(4) = 0.0625$$

(b) *Let N be the number of heads in 4 coin flips. Then N∼4.Intuitively, what is the expected value of N? Explain the reasoning behind your intuition.*

Solution
Looking at it intuitively, if you were to flip 4 coins with a fair chance of heads or tails, you would expect to get 2 heads.
(c) Compute the expected value of N, using the definition E[N]=$\Sigma n * p(n)$.

Solution
The expected value of N is 1*0.25+2*0.375+3*0.25+4*0.0625 = 2
(d) *Compute the variance of N, using the definition Var[N]=E[(NE[N])2].*

Solution

$$Var[N] = E[(N-2)^2] = (0-2)^2*0.0625+(1-2)^2*0.25+(2-2)^2*0.375+(3-2)^2*0.25+(4-2)^2*0.0625 = 1$$

Part 2

(a) *Write the probability distribution p4,k(n) for getting n heads on 4 coin flips, where the coin is a biased coin that lands on heads with probability k.*

Solution

$$(1-k)^4 + 4*k(1-k)^3 + 6*k^2(1-k)^2 + 4*k^3(1-k) + k^4$$

(b) *Let N be the number of heads in 4 coin flips of a biased coin. Then Np4,k. Intuitively, what is the expected value of N? Your answer should be in terms of k. Explain the reasoning behind your intuition.*

Solution

Intuitively, you would expect k times 4 number of heads, as if you were to substitute 0.5 in for k, you would get 2, which is the answer for 1b.

(c) *Compute the expected value of N, using the definition E[N]=np(n).*

Solution

$$0*((1-k)^4)+1*(4*k(1-k)^3)+2*(6*k^2(1-k)^2)+3*(4*k^3(1-k))+4*(k^4)$$

$$= (4k(1-3k+3k^2-k^3) + 2*(6*k^2(1-2k+k^2)) + 3*(4k^3-4k^4) + 4k^4$$

$$= 4k - 12k^2 + 12k^3 - 4k^4 + 2*(6k^2 - 12k^3 + 6k^4) + 12k^3 - 12k^4 + 4k^4$$

$$= 4k - 12k^2 + 12k^3 - 4k^4 + 12k^2 - 24k^3 + 12k^4 + 12k^3 - 12k^4 + 4k^4$$

$$= 4k$$

Matches guess in 2b