Assignment 33

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33-1

System of Equations:

$$ax + b = ln\left(\frac{1}{y} - 1\right)$$

$$a + b = ln\left(\frac{1}{0.2} - 1\right)$$

$$2a + b = ln\left(\frac{1}{0.25} - 1\right)$$

$$3a + b = ln\left(\frac{1}{0.5} - 1\right)$$

Matrix Equation:

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} ln(4) \\ ln(3) \\ ln(1) \end{bmatrix}$$

Approximation:

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} ln(4) \\ ln(3) \\ ln(1) \end{bmatrix}$$

$$\begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3.58 \\ 2.48 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} & -1 \\ -1 & \frac{7}{3} \end{bmatrix} \begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -1 \\ -1 & \frac{7}{3} \end{bmatrix} \begin{bmatrix} 3.58 \\ 2.48 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -0.69 \\ 2.2 \end{bmatrix}$$

Solution:

$$y = \frac{1}{1 + e^{-0.69x+2.2}}$$

Graph:

graph.png

33-2

(a)
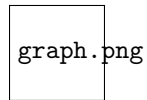
$$E[aX] = \int_{-\infty}^{\infty} x(a \cdot p(x))\,dx$$
$$= \int_{-\infty}^{\infty} a(x \cdot p(x))\,dx$$
$$= aE[X]$$

(b)

$$E[X_1 + X_2] = \int_{-\infty}^{\infty} x(p_1(x) + p_2(x))\,dx$$
$$= \int_{-\infty}^{\infty} x \cdot p_1(x) + x \cdot p_2(x)\,dx$$
$$= E[X_1] + E[X_2]$$

(c)

$$Var[X] = E[(X - E[X])^2]$$
$$= E[X^2 - 2X \cdot E[X] + E[X]^2]$$
$$=$$